of the system, Δa, above p_{amb} and T_{sat}. In this isobaric case:

$$\Delta a = (h_0 - h_f) - T_{sat}(s_0 - s_f) \quad (19)$$

but c_p should be nearly constant in the temperature range of interest, so:

$$\Delta a = c_p \left(\Delta\tau - T_{sat} \ln \frac{T_0}{T_{sat}}\right) \quad (20)$$

Since T_0/T_{sat} is slightly greater than unity,

$$\ln (T_0/T_{sat}) \simeq \left(\frac{T_0}{T_{sat}} - 1\right) - \frac{1}{2}\left(\frac{T_0}{T_{sat}} - 1\right)^2 \quad (21)$$

whence:

$$\Delta a = \frac{c_p}{2\, T_{sat}} \Delta\tau^2 \quad (22)$$

The quantity Δa specifies the capacity of a unit mass of superheated liquid for disrupting the system when nucleation is triggered. That Δa increases with the square of the superheat, shows for example why bumping in a smooth test tube is far more violent than boiling in a rough tea-kettle even though it occurs at only slightly higher superheats.

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FURTHER CALCULATIONS ON THE HEAT TRANSFER WITH TURBULENT FLOW BETWEEN PARALLEL PLATES

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NOMENCLATURE

y_0, half parallel plate gap;
y^+, dimensionless y co-ordinate;
ϵ_H, eddy diffusivity for heat;
ϵ_m, eddy diffusivity for momentum;
β, ratio of heat inputs at walls.

INTRODUCTION

The calculations described in [1] have been repeated with the object of determining the order of change in the heat-transfer results caused by different choices of certain basic assumptions in the analysis. In addition the calculations have been extended to a wider range of Prandtl numbers.

In the previous work [1] it was assumed that the eddy diffusivity of momentum was constant over the middle half of the passage (i.e. between $y_0^+/2$ and $3y_0^+/2$). That is, constant at the maximum value as given by Deissler's form of the eddy diffusivity variation. A reconsideration of the available experimental work, particularly that of Corcoran *et al.*, referred to in [1], showed that a more realistic assumption is to take the eddy diffusivity constant over the middle third of the passage (i.e. between $2y_0^+/3$ and $4y_0^+/3$). This modification was made but, in fact, has a negligible effect on any of the heat-transfer results.

It was also assumed in [1] that the ratio of the eddy diffusivities for momentum and heat was unity. In this extension the ratio has been calculated from the expressions proposed by Azer and Chao [2].

Results were given in the previous article for Prandtl numbers of 0·1, 1·0 and 10. This work includes the further eigenvalues for Prandtl numbers 0·01 and 0·7 and some information on the fully developed situation for Prandtl numbers 100 and 1000.

The large number of possible combinations of choice of assumption implies that only limited thermal boundary conditions could be examined. The case of a uniform heat flux on one side, the other being insulated ($\beta = 0$) was chosen. By superposition, the results were obtained for the cases of equal heat input on each side ($\beta = -1$) and equal heat input and output on each side ($\beta = 1$).

THE RELATIONSHIP BETWEEN ϵ_H and ϵ_m

A survey of the experimental and theoretical work on the relation between ϵ_H and ϵ_m revealed no consistent picture. One published theoretical relation which seems to predict the correct trends in the available experimental work is that due to Azer and Chao [2].

Their full expressions are complex but they suggested simpler forms which they fitted empirically to their theoretical equations. These are:

For fluids of Prandtl number 0·6 → 15

$$\frac{\epsilon_H}{\epsilon_m} = \frac{1 + 135\, Re^{-0\cdot45} \exp[-(y/y_0)^{0\cdot25}]}{1 + 57 Re^{-0\cdot46}\, Pr^{-0\cdot58} \exp[-(y/y_0)^{0\cdot25}]}$$

For $Pr < 0\cdot6$

$$\frac{\epsilon_H}{\epsilon_m} = \frac{1 + 135\, Re^{-0\cdot45} \exp[-(y/y_0)^{0\cdot25}]}{1 + 380\, (Re\, Pr)^{-0\cdot58} \exp[-(y/y_0)^{0\cdot25}]} \qquad (1)$$

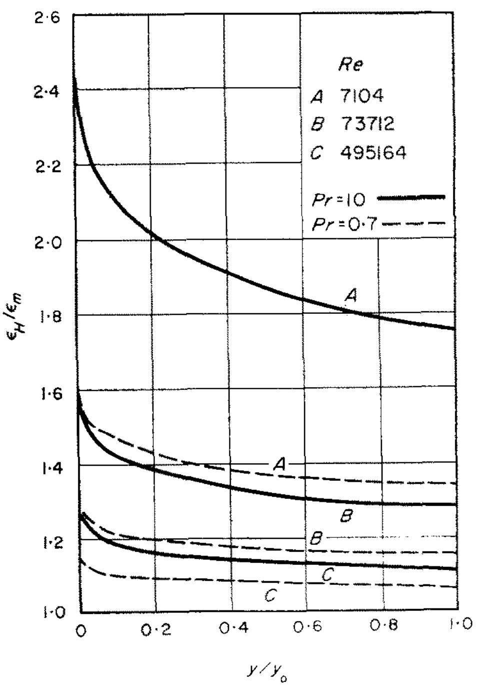

FIG 1(a). ϵ_H/ϵ_m relationship [2]. $Pr > 0\cdot6$.

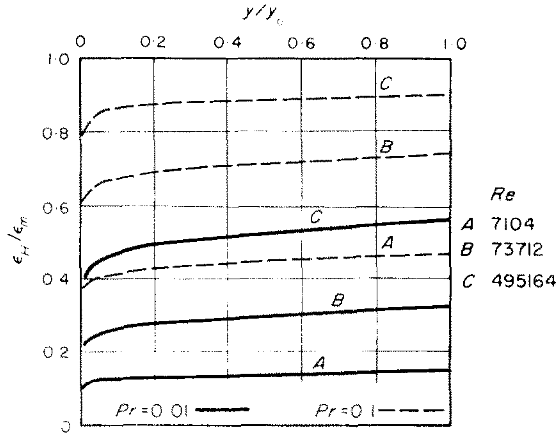

FIG. 1(b). ϵ_H/ϵ_m relationship [2]. $Pr < 0\cdot6$.

Figure 1 shows these relationships plotted for the arbitrary values of Reynolds number and Prandtl number which were used in this work. The odd values of Reynolds number are the result of choosing whole number values of y_0^+ (i.e. 126, 926, 5026).

An earlier theoretical relationship has been proposed by Jenkins [3] and this is not restricted to Prandtl numbers less than 15. It has been used, with a certain modification, by Leung, Kays and Reynolds [4] in their study of the annulus.

RESULTS

A reconsideration of the work of [1] showed that good accuracy is obtainable with fewer eigenvalues than the seven which were then calculated. In fact, it appears that the first four will suffice for $x/d > 1$.

Table 1 lists eigenvalues and constants corresponding to those given in [1] and assuming ϵ_m is constant over the middle third of the duct. These values apply to the case of uniform heat flux on one side, the other being adiabatic. The calculations were carried out on the Manchester University Atlas computer.

From these values one can obtain fully developed Nusselt numbers and entrance region values for any combination of surface heat fluxes and for axially varying heat fluxes such as those considered in [1].

Table 2 lists Nusselt numbers for the fully developed situation with the different symmetrical and unsymmetrical boundary conditions previously mentioned.

Figure 2 shows a typical entrance region variation at the Reynolds number of 73712 and as would be expected from Fig. 1, the inclusion of Azer and Chao's expressions cause the Nusselt numbers to lie higher for $Pr > 0\cdot6$ and lower for $Pr < 0\cdot6$ than with the assumption $\epsilon_H/\epsilon_m = 1$. Figure 2 again shows that unsymmetrical boundary conditions may greatly increase the thermal entrance length but, in this connection, the inclusion of Azer and Chao's relations does not have much effect. Also, one may conclude that the order of magnitude of

Table 1. Eigenvalues and constants. Uniform heat flux on one side, the other side insulated

$Pr = 0{\cdot}01$

		ϵ_H/ϵ_m = equation (1)		$\epsilon_H/\epsilon_m = 1$	
$Re = 7104$	G_i	0·1736444		0·1638712	
	G_o	−0·073660		−0·0683918	
	n	λ_n	C_n	λ_n	C_n
	1	23·9424	0·527983	24·8619	0·515349
	2	46·7620	0·148656	48·5953	0·149300
	3	69·6821	0·070088	72·3150	0·069632
	4	92·5428	0·0414758	95·8611	0·044383
$Re = 73712$	G_i	0·145843		0·1094899	
	G_o	−0·063858		−0·0453442	
	n	λ_n	C_n	λ_n	C_n
	1	25·7198	0·544652	30·5836	0·515362
	2	50·3284	0·154201	60·0596	0·151632
	3	74·8007	0·074079	88·6962	0·076569
	4	99·0004	0·043749	116·655	0·048509
$Re = 495164$	G_i	0·068127		0·0458873	
	G_o	−0·025451		−0·0161960	
	n	λ_n	C_n	λ_n	C_n
	1	41·2424	0·472520	51·6839	0·443927
	2	80·2075	0·151901	101·447	0·143901
	3	117·146	0·085386	147·600	0·085061
	4	152·682	0·054414	191·683	0·056576

$Pr = 0{\cdot}1$

$Re = 7104$	G_i	0·135142		0·107053	
	G_o	−0·052857		−0·038432	
	n	λ_n	C_n	λ_n	C_n
	1	8·9746	0·489911	10·5851	0·453048
	2	17·5206	0·144726	20·5723	0·140911
	3	25·9532	0·073771	30·3202	0·076073
	4	34·1919	0·046151	39·7985	0·050089
$Re = 73712$	G_i	0·040448		0·031188	
	G_o	−0·012823		−0·0090897	
	n	λ_n	C_n	λ_n	C_n
	1	18·4530	0·403885	22·0430	0·376900
	2	35·9032	0·136962	42·2838	0·133795
	3	52·1180	0·083628	61·3268	0·080998
	4	67·4772	0·056734	79·7950	0·055965
$Re = 495164$	G_i	0·009349		0·008391	
	G_o	−0·002321		−0·001882	
	n	λ_n	C_n	λ_n	C_n
	1	44·2598	0·310840	48·7477	0·294157
	1	86·2024	0·114384	93·1388	0·112429
	3	123·869	0·078418	134·036	0·073979
	4	159·094	0·057646	173·328	0·054151

Table 1 continued

$Pr = 0{\cdot}7$

		ϵ_H/ϵ_m = equation (1)		$\epsilon_H/\epsilon_m = 1$	
$Re = 7104$	G_i	0·0376907		0·0448027	
	G_o	−0·008001		−0·0104563	
	n	λ_n	C_n	λ_n	C_n
	1	8·8736	0·273419	7·7628	0·292106
	2	17·4189	0·101452	15·1606	0·119184
	3	25·1683	0·076028	21·9330	0·072589
	4	32·1251	0·064565	28·0676	0·072905
$Re = 73712$	G_i	0·007402		0·00837698	
	G_o	−0·001363		−0·00058292	
	n	λ_n	C_n	λ_n	C_n
	1	21·6586	0·230823	20·1239	0·237755
	2	42·5354	0·082445	39·3246	0·088104
	3	61·3990	0·056980	56·6776	0·059732
	4	78·9602	0·043472	72·8302	0·047235
$Re = 495164$	G_i	0·001715		0·0018539	
	G_o	−0·000341		−0·0003627	
	n	λ_n	C_n	λ_n	C_n
	1	48·0596	0·203928	46·3359	0·204222
	2	94·2035	0·074603	90·5511	0·076151
	3	135·427	0·052449	130·029	0·053641
	4	173·824	0·039844	166·792	0·041173

$Pr = 1$

$Re = 7104$	G_i	0·030283		0·0369355	
	G_o	−0·005490		−0·007410	
	n	λ_n	C_n	λ_n	C_n
	1	8·9882	0·234815	7·8029	0·263048
	2	17·6568	0·090674	15·0001	0·104846
	3	25·4370	0·072482	21·6133	0·078378
	4	32·3297	0·065204	27·6359	0·066777
$Re = 73712$	G_i	0·005758		0·006597	
	G_o	−0·000944		−0·001035	
	n	λ_n	C_n	λ_n	C_n
	1	21·8895	0·203418	20·9064	0·205371
	2	43·0269	0·072887	39·9055	0·080378
	3	62·1062	0·050856	57·4954	0·054798
	4	79·8534	0·039264	74·2859	0·042367
$Re = 495164$	G_i	0·00134896		0·001468	
	G_o	−0·0001883		−0·0002059	
	n	λ_n	C_n	λ_n	C_n
	1	48·3232	0·185898	48·2497	0·167790
	2	94·7660	0·067951	92·0672	0·066072
	3	136·250	0·047838	132·2351	0·045081
	4	174·886	0·036425	170·6821	0·034155

Table 1 continued

$Pr = 10$

		ϵ_H/ϵ_m = equation (1)		$\epsilon_H/\epsilon_m = 1$	
$Re = 7104$	G_i	0·009844		0·012909	
	G_o	−0·000481		−0·000905	
	n	λ_n	C_n	λ_n	C_n
	1	9·7418	0·065868	7·4609	0·090942
	2	19·2193	0·032237	14·2449	0·048317
	3	27·4355	0·038249	20·1805	0·053720
	4	34·2548	0·052047	25·1755	0·065565
$Re = 73712$	G_i	0·0016301		0·00192814	
	G_o	−0·0000846		−0·000111	
	n	λ_n	C_n	λ_n	C_n
	1	22·7786	0·067715	20·7885	0·072723
	2	44·9276	0·024597	39·6525	0·029597
	3	64·8847	0·017848	57·0367	0·021372
	4	83·4198	0·014539	73·6425	0·017723
$Re = 495164$	G_i	0·0003475		0·0003832	
	G_o	0·00001312		−0·00001691	
	n	λ_n	C_n	λ_n	C_n
	1	49·2320	0·083752	48·1997	0·062193
	2	96·7114	0·030519	91·9585	0·026276
	3	139·117	0·021584	132·0501	0·018122
	4	178·603	0·016577	170·4069	0·013909

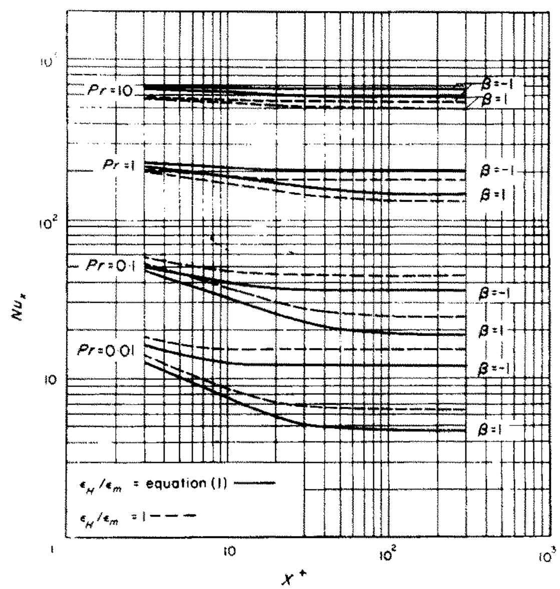

FIG. 2. Local Nusselt number variation. $Re = 73712$.

Table 2. Fully developed Nusselt numbers

	Re / β	ϵ_H/ϵ_m = equation (1) Nu_∞			$\epsilon_H/\epsilon_m = 1$ Nu_∞		
		7104	73712	495164	7104	73712	495164
$Pr = 0.01$	1	4·04	4·77	10·69	4·31	6·46	16·11
	0	5·76	6·86	14·67	6·10	9·30	21·78
	−1	10·00	12·20	23·43	10·47	15·59	33·68
$Pr = 0.1$	1	5·32	18·77	85·69	6·70	24·14	95·98
	0	7·40	24·75	107·0	9·25	31·50	119·8
	−1	12·15	36·20	142·3	14·50	45·18	159·4
$Pr = 0.7$	1	21·89	114·1	486·3	18·10	111·6	451·1
	0	26·50	135·1	583·0	22·30	119·4	540·0
	−1	33·68	165·6	727·6	29·12	128·3	670·6
$Pr = 1$	1	27·95	149·2	650·6	22·23	128·7	597·2
	0	33·0	174·0	741·3	26·82	150·5	680·8
	−1	40·34	207·7	861·6	33·76	181·1	791·9
$Pr = 10$	1	96·86	583·2	2773	71·98	490·4	2499
	0	101·5	613·4	2878	76·90	518·6	2609
	−1	106·8	647·0	2900	82·45	550·6	2732
$Pr = 100$	1				161·3	1178	6298
	0				165·6	1192	6345
					168·6	1206	6394
$Pr = 1000$	1				308·9	2270	12,310
	0				310·6	2290	12,420
	−1				314·1	2329	12,500

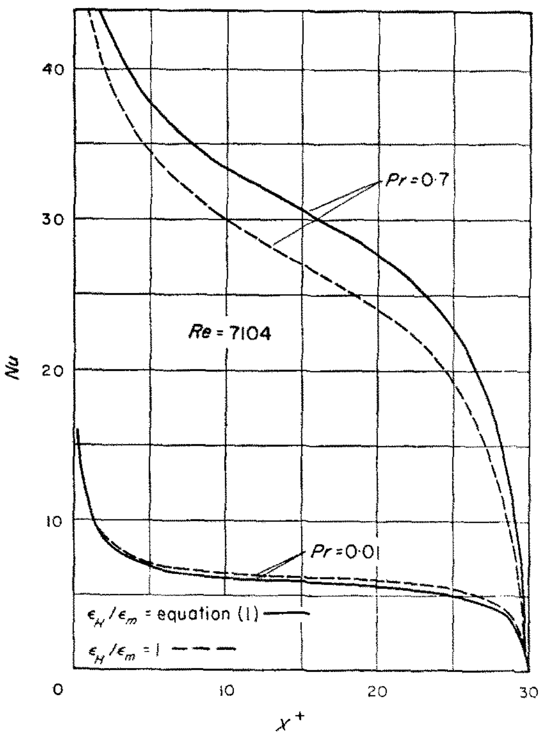

FIG. 3. *Nu* variation for sinusoidal heat input.

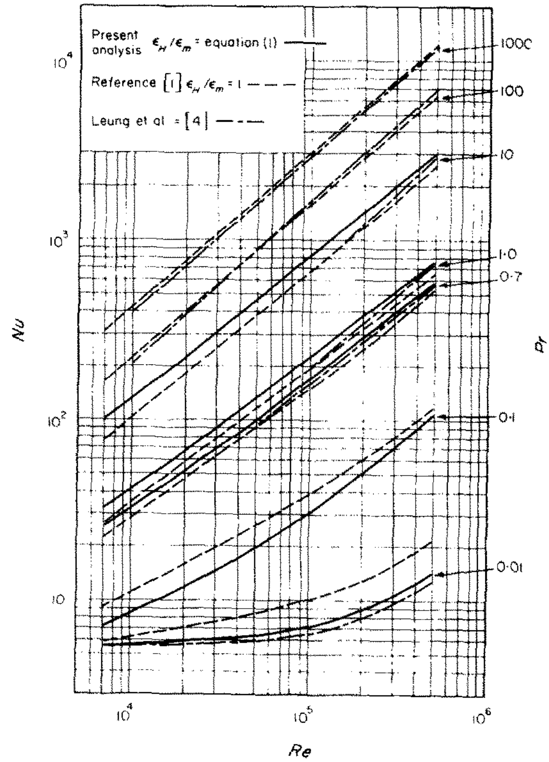

FIG. 4. Nu_∞–Re relationships, $\beta = 0$.

change caused by the expressions is much the same for both symmetrical and unsymmetrical heating.

Figure 3 gives a result similar to one in the previous article for a sinusoidal heat flux distribution along one side (the duct length being 30 *d*).

Figure 4 shows a number of results for the fully developed situation and it is here possible to make comparisons with the work of Leung *et al.* [4]. They modified Jenkins's expression, which originally gave $\epsilon_H/\epsilon_m = 1$ for $Pr = 1$, by including a multiplying factor of 1·2 to give $\epsilon_H/\epsilon_m = 1{\cdot}2$ at $Pr = 1$. Figure 4 shows that the calculations of Leung *et al.* for $Pr = 0{\cdot}7$ agree fairly closely with this work taking $\epsilon_H/\epsilon_m = 1$. For low Prandtl numbers their results agree with those of this work when Azer and Chao's expression is included. Azer and Chao did not give any experimental evidence in support of their expression for the higher Prandtl numbers, whereas Leung *et al.* showed that Jenkins's form is justified quite well for $Pr = 0{\cdot}7$ in the annulus.

For $Pr = 0{\cdot}1$ Leung *et al.* give no results and comparison is shown between the results with and without Azer and Chao's expression.

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